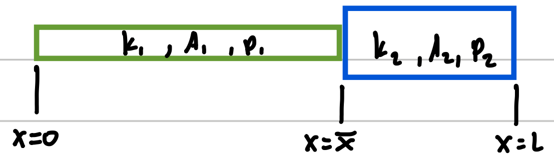


$$\alpha_i^2 = \frac{h\,p_i}{k_i A_i} = \frac{h 2\pi r}{k_i \pi r^2} = \frac{2h}{k_i r} \Rightarrow \alpha_i = \sqrt{\frac{2h}{k_i r}}$$

Analytical

$$T_1(x) = C_1 \sinh(\alpha_1 x) + D_1 \cosh(\alpha_1 x) \qquad 0 \le x \le \bar{x} \qquad T'(x) = C_1 \alpha_1 \cosh(\alpha_1 x) + D_1 \alpha_1 \sinh(\alpha_1 x)$$

$$T_2(x) = C_2 \sinh(\alpha_2 x) + D_2 \cosh(\alpha_2 x) \qquad \bar{x} \le x \le L \qquad T'(x) = C_2 \alpha_2 \cosh(\alpha_2 x) + D_2 \alpha_2 \sinh(\alpha_2 x)$$

Case 1

1) $T_1(0) = 0$, 2) $T_2(L) = 100$

3) $T_1(\bar{x}) = T_2(\bar{x})$ 4) $k_1 A_1 T_1'(\bar{x}) = k_2 A_2 T_2'(\bar{x})$

1) $T_1(0) = \boxed{D_1 = 0}$

2) $\boxed{\sinh(\alpha_2 L)\, C_2 + \cosh(\alpha_2 L)\, D_2 = 100}$

3) $C_1 \sinh(\alpha_1 \bar{x}) = C_2 \sinh(\alpha_2 \bar{x}) + D_2 \cosh(\alpha_2 \bar{x})$

$\boxed{\sinh(\alpha_1 \bar{x})\, C_1 - \sinh(\alpha_2 \bar{x})\, C_2 - \cosh(\alpha_2 \bar{x})\, D_2 = 0}$

4) $k_1 A_1 [\alpha_1 C_1 \cosh(\alpha_1 \bar{x})] = k_2 A_2 [\alpha_2 C_2 \cosh(\alpha_2 \bar{x}) + \alpha_2 D_2 \sinh(\alpha_2 \bar{x})]$

$k_1 A_1 \alpha_1 \cosh(\alpha_1 \bar{x})\, C_1 - k_2 A_2 \alpha_2 \cosh(\alpha_2 \bar{x})\, C_2 - k_2 A_2 \alpha_2 \sinh(\alpha_2 \bar{x})\, D_2 = 0$

$\frac{k_1 A_1 \alpha_1}{k_2 A_2 \alpha_2} \cosh(\alpha_1 \bar{x})\, C_1 - \cosh(\alpha_2 \bar{x})\, C_2 - \sinh(\alpha_2 \bar{x})\, D_2 = 0$

let $\mathcal{E} = \frac{k_1 A_1 \alpha_1}{k_2 A_2 \alpha_2}$ $\quad \boxed{\mathcal{E} \cosh(\alpha_1 \bar{x})\, C_1 - \cosh(\alpha_2 \bar{x})\, C_2 - \sinh(\alpha_2 \bar{x})\, D_2 = 0}$

Case 2

2, 3, + 4 BC's are same

1) $k_1 A_1 T_1'(0) = 0$

$\alpha C_1 = 0$

$\boxed{C_1 = 0}$

2) $\boxed{\sinh(\alpha_2 L) C_2 + \cosh(\alpha_2 L) D_2 = 100}$

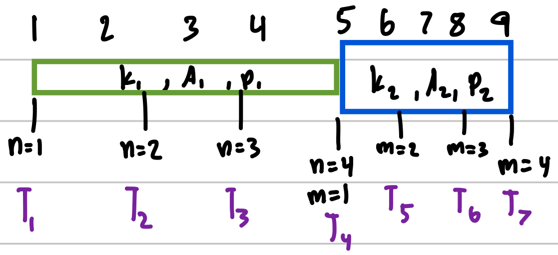
3) $D_1 \cosh(\alpha_1 \bar{x}) = C_2 \sinh(\alpha_2 \bar{x}) + D_2 \cosh(\alpha_2 \bar{x})$

$\boxed{\cosh(\alpha_1 \bar{x}) D_1 - \sinh(\alpha_2 \bar{x}) C_2 - \cosh(\alpha_2 \bar{x}) D_2 = 0}$

4) $k_1 A_1 [\alpha_1 D_1 \sinh(\alpha_1 \bar{x})] = k_2 A_2 [\alpha_2 C_2 \cosh(\alpha_2 \bar{x}) + \alpha_2 D_2 \sinh(\alpha_2 \bar{x})]$

$\boxed{\varepsilon \sinh(\alpha_1 \bar{x}) D_1 - \cosh(\alpha_2 \bar{x}) C_2 - \sinh(\alpha_2 \bar{x}) D_2 = 0}$

FDM



Case 1

Centered FDM, $n=2,3 \quad m=2,3$

$$T_n'' = \frac{T_{n+1} - 2T_n + T_{n-1}}{\Delta x^2}$$

ODE $\quad T_n'' - \alpha^2 T_n = 0 \qquad n \in [1, N]$

$$\frac{T_{n+1} - 2T_n + T_{n-1}}{\Delta x^2} - \alpha^2 T_n = 0$$

$$T_{n-1}\left(\frac{1}{\Delta x^2}\right) + T_n\left(\frac{-2}{\Delta x^2} - \alpha^2\right) + T_{n+1}\left(\frac{1}{\Delta x^2}\right) = 0$$

$$T_{n-1} + \left(-2 - \alpha^2 \Delta x^2\right) T_n + T_{n+1} = 0$$

let $k_i = 2 + \alpha_i^2 \Delta x_i^2$

$$\boxed{-T_{n-1} + kT_n - T_{n+1} = 0}$$

$$\boxed{T_1 = 0 \,,\; T_7 = 100}$$

$-T_1 + k_1 T_2 - T_3 = 0 \Rightarrow k_1 T_2 - T_3 = 0$

$-T_2 + k_1 T_3 - T_4 = 0$

$-T_3 + k T_4 - T_5 = 0$ not allowed because we cross the boundary

$-T_4 + k_2 T_5 - T_6 = 0$

$-T_5 + k_2 T_6 - T_7 = 0 \Rightarrow -T_5 + k_2 T_6 = 100$

use $\dot{Q}(n=4) = -\dot{Q}(m=1)$

$-k_1 A_1 T_1'(n=4) = k_2 A_2 T_2'(m=1)$

let $T_{1,4}'$ $\qquad$ let $T_{2,4}'$

$T'_{1,4}$ uses backwards Taylor Series

$$T_3 = T_4 - \Delta x_1 T'_{1,4} + \frac{\Delta x_1^2}{2} T''_{1,4}$$

$$\Delta x_1 T'_{1,4} = T_4 - T_3 + \frac{\Delta x_1^2}{2}\left(+\alpha^2 T_4\right)$$ from ODE

$$T'_{1,4} = \frac{T_4 - T_3}{\Delta x_1} + \frac{\Delta x_1 \alpha_1^2}{2} T_4 = -\frac{1}{\Delta x_1} T_3 + \left(\frac{1}{\Delta x_1} + \frac{\Delta x_1 \alpha^2}{2}\right) T_4$$

$T'_{2,4}$ uses forward Taylor Series

$$T_5 = T_4 + \Delta x_2 T'_{2,4} + \frac{\Delta x_2^2}{2} T''_{2,4}$$

$$\Delta x_2 T'_{2,4} = T_5 - T_4 - \frac{\Delta x_2^2}{2}\left(+\alpha_2^2 T_4\right)$$

$$T'_{2,4} = \frac{T_5 - T_4}{\Delta x_2} - \frac{\Delta x_2^2}{2}\alpha_2^2 T_4 = \left(-\frac{1}{\Delta x_2} - \frac{\Delta x_2^2 \alpha_2^2}{2}\right) T_4 + \frac{1}{\Delta x_2} T_5$$

$$k_1 A_1 \left[-\frac{1}{\Delta x_1} T_3 + \left(\frac{1}{\Delta x_1} + \frac{\Delta x_1 \alpha_1^2}{2}\right) T_4\right] = -k_2 A_2 \left[\left(-\frac{1}{\Delta x_2} - \frac{\Delta x_2^2 \alpha_2^2}{2}\right) T_4 + \frac{1}{\Delta x_2} T_5\right]$$

$$\left(-\frac{k_1 A_1}{\Delta x_1}\right) T_3 + \underbrace{\left(\frac{k_1 A_1}{\Delta x_1} - \frac{k_2 A_2}{\Delta x_2} + \frac{k_1 A_1 \Delta x_1 \alpha_1^2}{2} - \frac{k_2 A_2 \Delta x_2 \alpha_2^2}{2}\right)}_{= \beta_1 - \beta_2 - \frac{k_1 A_1 \Delta x_1 \alpha_1^2}{2} + \frac{k_2 A_2 \Delta x_2 \alpha_2^2}{2} = \rho} T_4 + \left(\frac{k_2 A_2}{\Delta x_2}\right) T_5 = 0$$

$$\frac{k_1 A_1}{\Delta x_1} = \beta_1 \qquad \frac{k_2 A_2}{\Delta x_2} = \beta_2$$

$$\begin{bmatrix} k_1 & -1 & 0 & 0 & 0 \\ -1 & k_1 & -1 & 0 & 0 \\ 0 & -\beta_1 & \rho & \beta_2 & 0 \\ 0 & 0 & -1 & k_2 & -1 \\ 0 & 0 & 0 & -1 & k_2 \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} T_1 = 0 \\ 0 \\ 0 \\ 0 \\ T_7 = 100 \end{bmatrix}$$

Case 2

can't use $T_1 = 0$. instead, $T_1' = 0$

backwards Taylor Series at T_2

$$T_2 = T_1 + \Delta x_1 T_1' + \frac{\Delta x_1^2}{2} T_1''$$

$$\Delta x_1 T_1' = T_2 - T_1 - \frac{\Delta x_1^2}{2} T_1''$$

$$T_1' = \frac{T_2 - T_1}{\Delta x_1} - \frac{\Delta x_1}{2}\left(+\alpha_1^2 T_1\right)$$

$$\left(\frac{-1}{\Delta x_1} - \frac{\alpha_1^2 \Delta x_1}{2}\right) T_1 + \frac{1}{\Delta x_1} T_2 = 0$$

$$\left(+1 + \frac{\alpha_1^2 \Delta x_1^2}{2}\right) T_1 + T_2 = 0$$

let $k' = \frac{k}{2} = \frac{1 + \alpha_1^2 \Delta x_1^2}{2}$

$$\begin{bmatrix} k_1' & -1 & 0 & 0 & 0 & 0 \\ -1 & k_1 & -1 & 0 & 0 & 0 \\ 0 & -1 & k_1 & -1 & 0 & 0 \\ 0 & 0 & -\beta_1 & \beta & -\beta_2 & 0 \\ 0 & 0 & 0 & -1 & k_2 & -1 \\ 0 & 0 & 0 & 0 & -1 & k_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} T_1' = 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_7 = 100 \end{bmatrix}$$

* ONLY CASE 1

Error of junction temperature

FEM

Case 1