

$$\alpha_i^2 = \frac{h p_i}{k_i A_i} = \frac{h 2\pi r}{k_i \pi r^2} = \frac{2h}{k_i r} \Rightarrow \alpha_i = \sqrt{\frac{2h}{k_i r}}$$

Analytical

$$T_1(x) = C_1 \sinh(\alpha_1 x) + D_1 \cosh(\alpha_1 x) \quad 0 \leq x \leq \bar{x} \quad T'(x) = C_1 \alpha_1 \cosh(\alpha_1 x) + D_1 \alpha_1 \sinh(\alpha_1 x)$$

$$T_2(x) = C_2 \sinh(\alpha_2 x) + D_2 \cosh(\alpha_2 x) \quad \bar{x} \leq x \leq L \quad T'(x) = C_2 \alpha_2 \cosh(\alpha_2 x) + D_2 \alpha_2 \sinh(\alpha_2 x)$$

Case 1

$$\begin{aligned} 1) & T_1(0) = 0, & 2) & T_1(L) = 100 \\ 3) & T_1(\bar{x}) = T_2(\bar{x}) & 4) & k_1 A_1 T'_1(\bar{x}) = k_2 A_2 T'_2(\bar{x}) \end{aligned}$$

$$1) \quad T_1(0) = D_1 = 0$$

$$2) \quad \sinh(\alpha_2 L) C_2 + \cosh(\alpha_2 L) D_2 = 100$$

$$3) \quad C_1 \sinh(\alpha_1 \bar{x}) = C_2 \sinh(\alpha_2 \bar{x}) + D_2 \cosh(\alpha_2 \bar{x})$$

$$\sinh(\alpha_1 \bar{x}) C_1 - \sinh(\alpha_2 \bar{x}) C_2 - \cosh(\alpha_2 \bar{x}) D_2 = 0$$

$$4) \quad k_1 A_1 [\alpha_1 C_1 \cosh(\alpha_1 \bar{x})] = k_2 A_2 [\alpha_2 C_2 \cosh(\alpha_2 \bar{x}) + \alpha_2 D_2 \sinh(\alpha_2 \bar{x})]$$

$$k_1 A_1 \alpha_1 \cosh(\alpha_1 \bar{x}) C_1 - k_2 A_2 \alpha_2 \cosh(\alpha_2 \bar{x}) C_2 - k_2 A_2 \alpha_2 \sinh(\alpha_2 \bar{x}) D_2 = 0$$

$$\frac{k_1 A_1 \alpha_1 \cosh(\alpha_1 \bar{x}) C_1 - \cosh(\alpha_2 \bar{x}) C_2 - \sinh(\alpha_2 \bar{x}) D_2 = 0}{k_2 A_2 \alpha_2}$$

$$\text{let } \mathcal{E} = \frac{k_1 A_1 \alpha_1}{k_2 A_2 \alpha_2} \quad \mathcal{E} \cosh(\alpha_1 \bar{x}) C_1 - \cosh(\alpha_2 \bar{x}) C_2 - \sinh(\alpha_2 \bar{x}) D_2 = 0$$

Case 2

2, 3, & 4 BCs are same

$$1) k_1 A_1 T'_1(0) = 0$$

$$\alpha L_1 = 0$$

$$C_1 = 0$$

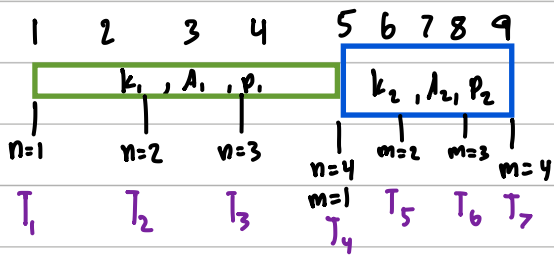
$$2) \sinh(\alpha_2 L) C_2 + \cosh(\alpha_2 L) D_2 = 100$$

$$3) \begin{aligned} D_1 \cosh(\alpha_1 \bar{x}) &= C_2 \sinh(\alpha_2 \bar{x}) + D_2 \cosh(\alpha_2 \bar{x}) \\ \cosh(\alpha_1 \bar{x}) D_1 - \sinh(\alpha_2 \bar{x}) C_2 - \cosh(\alpha_2 \bar{x}) D_2 &= 0 \end{aligned}$$

$$4) k_1 A_1 [\alpha_1 D_1 \sinh(\alpha_1 \bar{x})] = k_2 A_2 [\alpha_2 C_2 \cosh(\alpha_2 \bar{x}) + \alpha_2 D_2 \sinh(\alpha_2 \bar{x})]$$

$$E \sinh(\alpha_1 \bar{x}) D_1 - \cosh(\alpha_2 \bar{x}) C_2 - \sinh(\alpha_2 \bar{x}) D_2 = 0$$

FDM



Case 1

Centered FDM, $n=2,3$ $m=2,3$

$$T_n'' = \frac{T_{n+1} - 2T_n + T_{n-1}}{\Delta x^2}$$

ODE $T_n'' - \alpha^2 T_n = 0 \quad n \in [1, N]$

$$\frac{T_{n+1} - 2T_n + T_{n-1}}{\Delta x^2} - \alpha^2 T_n = 0$$

$$T_{n-1} \left(\frac{1}{\Delta x^2} \right) + T_n \left(\frac{-2}{\Delta x^2} - \alpha^2 \right) + T_{n+1} \left(\frac{1}{\Delta x^2} \right) = 0$$

$$T_{n-1} + (-2 - \alpha^2 \Delta x^2) T_n + T_{n+1} = 0$$

let $k_i = 2 + \alpha^2 \Delta x_i^2$

$$-T_{n-1} + k T_n - T_{n+1} = 0$$

$$T_1 = 0, T_7 = 100$$

$$-T_1 + k_1 T_2 - T_3 = 0 \Rightarrow k_1 T_2 - T_3 = 0$$

$$-T_2 + k_1 T_3 - T_4 = 0$$

$$-T_3 + k_1 T_4 - T_5 = 0 \text{ not allowed because we cross the boundary}$$

$$-T_4 + k_2 T_5 - T_6 = 0$$

$$-T_5 + k_2 T_6 - T_7 = 0 \Rightarrow -T_5 + k_2 T_6 = 100$$

use $\dot{Q}(n=4) = -\dot{Q}(m=1)$

$$-k_1 A_1 T_1'(n=4) = k_2 A_2 T_2'(m=1)$$

let $T_{1,4}'$

let $T_{2,4}'$

$T'_{1,4}$ uses backwards Taylor Series

$$T_3 = T_4 - \Delta x_1 T'_{1,4} + \frac{\Delta x_1^2}{2} T''_{1,4}$$

$$\Delta x_1 T'_{1,4} = T_4 - T_3 + \frac{\Delta x_1^2}{2} (\alpha_1^2 T_4) \quad \text{from ODE}$$

$$T'_{1,4} = \frac{T_4 - T_3}{\Delta x_1} + \frac{\Delta x_1 \alpha_1^2}{2} T_4 = -\frac{1}{\Delta x_1} T_3 + \left(\frac{1}{\Delta x_1} + \frac{\Delta x_1 \alpha_1^2}{2} \right) T_4$$

$T'_{2,4}$ uses forward Taylor Series

$$T_5 = T_4 + \Delta x_2 T'_{2,4} + \frac{\Delta x_2^2}{2} T''_{2,4}$$

$$\Delta x_2 T'_{2,4} = T_5 - T_4 - \frac{\Delta x_2^2}{2} (\alpha_2^2 T_4)$$

$$T'_{2,4} = \frac{T_5 - T_4}{\Delta x_2} - \frac{\Delta x_2 \alpha_2^2}{2} T_4 = \left(-\frac{1}{\Delta x_2} - \frac{\Delta x_2 \alpha_2^2}{2} \right) T_4 + \frac{1}{\Delta x_2} T_5$$

$$k_1 A_1 \left[-\frac{1}{\Delta x_1} T_3 + \left(\frac{1}{\Delta x_1} + \frac{\Delta x_1 \alpha_1^2}{2} \right) T_4 \right] = -k_2 A_2 \left[\left(-\frac{1}{\Delta x_2} - \frac{\Delta x_2 \alpha_2^2}{2} \right) T_4 + \frac{1}{\Delta x_2} T_5 \right]$$

$$\left(-\frac{k_1 A_1}{\Delta x_1} \right) T_3 + \left(\frac{k_1 A_1}{\Delta x_1} - \frac{k_2 A_2}{\Delta x_2} + \frac{k_1 A_1 \Delta x_1 \alpha_1^2}{2} - \frac{k_2 A_2 \Delta x_2 \alpha_2^2}{2} \right) T_4 + \left(\frac{k_2 A_2}{\Delta x_2} \right) T_5 = 0$$

$$\frac{k_1 A_1}{\Delta x_1} = \beta_1 \quad \frac{k_2 A_2}{\Delta x_2} = \beta_2 \quad = \beta_1 - \beta_2 - \frac{k_1 A_1 \Delta x_1 \alpha_1^2}{2} + \frac{k_2 A_2 \Delta x_2 \alpha_2^2}{2} = \rho$$

$$\begin{bmatrix} k_1 & -1 & 0 & 0 & 0 \\ -1 & k_1 & -1 & 0 & 0 \\ 0 & -\beta_1 & \rho & \beta_2 & 0 \\ 0 & 0 & -1 & k_2 & -1 \\ 0 & 0 & 0 & -1 & k_2 \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} T_1 = 0 \\ 0 \\ 0 \\ 0 \\ T_7 = 100 \end{bmatrix}$$

Case 2

can't use $T_1=0$. instead, $T_1'=0$

backwards Taylor Series at T_2

$$T_2 = T_1 + \Delta x_1 T_1' + \frac{\Delta x_1^2}{2} T_1''$$

$$\Delta x_1 T_1' = T_2 - T_1 - \frac{\Delta x_1^2}{2} T_1''$$

$$T_1' = \frac{T_2 - T_1}{\Delta x_1} - \frac{\Delta x_1}{2} T_1''$$

$$\left(-\frac{1}{\Delta x_1} - \frac{\alpha_1^2 \Delta x_1}{2} \right) T_1 + \frac{1}{\Delta x_1} T_2 = 0$$

$$\left(+1 + \frac{\alpha_1^2 \Delta x_1^2}{2} \right) T_1 + T_2 = 0$$

$$\text{let } k' = \frac{k}{2} = \frac{1 + \alpha_1^2 \Delta x_1^2}{2}$$

$$\begin{bmatrix} k' & -1 & 0 & 0 & 0 & 0 \\ -1 & k_1 & -1 & 0 & 0 & 0 \\ 0 & -1 & k_1 & -1 & 0 & 0 \\ 0 & 0 & -\beta_1 & \rho & -\beta_2 & 0 \\ 0 & 0 & 0 & -1 & k_2 & -1 \\ 0 & 0 & 0 & 0 & -1 & k_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} T_1' = 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_7 = 100 \end{bmatrix}$$

* ONLY CASE 1

Error of junction temperature

FEM

Case 1